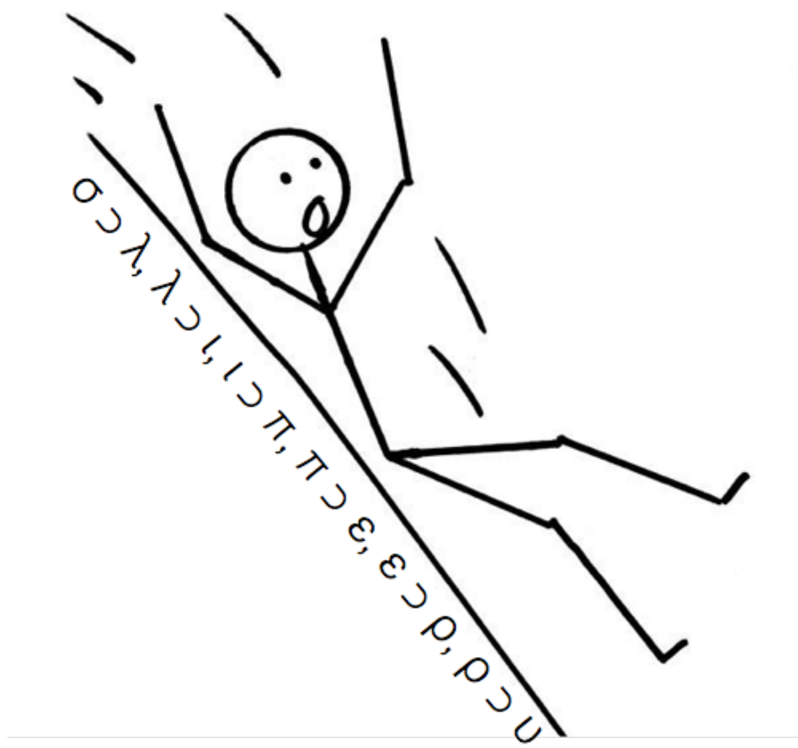
Introduction to Logic and Critical Thinking

Version 2.0



Matthew J. Van Cleave
vancleave@mac.com

2.11 Proofs and the 8 valid forms of inference

Although truth tables are our only formal method of deciding whether an argument is valid or invalid in propositional logic, there is another formal method of proving that an argument *is* valid: the method of proof. Although you cannot construct a proof to show that an argument is invalid, you can construct proofs to show that an argument is valid. The reason proofs are helpful, is that they allow us to show that certain arguments are valid much more efficiently than do truth tables. For example, consider the following argument:

1. (R v S) ⊃ (T ⊃ K)
2. ~K
3. R v S /∴ ~T

(Note: in this section I will be writing the conclusion of the argument to the right of the last premise—in this case premise 3. As before, the conclusion we are trying to derive is denoted by the "therefore" sign, "∴".) We could attempt to prove this argument is valid with a truth table, but the truth table would be 16 rows long because there are four different atomic propositions that occur in this argument, R, S, T, and K. If there were 5 or 6 different atomic propositions, the truth table would be 32 or 64 lines long! However, as we will soon see, we could also prove this argument is valid with only two additional lines. That seems a much more efficient way of establishing that this argument is valid. We will do this a little later—after we have introduced the 8 valid forms of inference that you will need in order to do proofs. Each line of the proof will be justified by citing one of these rules, with the last line of the proof being the conclusion that we are trying to ultimately establish. I will introduce the 8 valid forms of inference in groups, starting with the rules that utilize the horseshoe and negation.

The first of the 8 forms of inference is "**modus ponens**" which is Latin for "way that affirms." Modus ponens has the following form:

1. p ⊃ q
2. p
3. ∴ q

What this form says, in words, is that if we have asserted a conditional statement (p ⊃ q) and we have also asserted the antecedent of that conditional statement (p), then we are entitled to infer the consequent of that conditional statement (q). For example, if I asserted the conditional, "if it is raining, then the ground is wet" and I also asserted "it is raining" (the antecedent of that conditional) then I (or anyone else, for that matter) am entitled to assert the consequent of the conditional, "the ground is wet."

As with any valid forms of inference in this section, we can prove that modus ponens is valid by constructing a truth table. As you see from the truth table below, this argument form passes the truth table test of validity (since there is no row of the truth table on which the premises are all true and yet the conclusion is false).

p	q	p ⊃ q	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Thus, any argument that has this same form is valid. For example, the following argument also has this same form (modus ponens):

1. (A • B) ⊃ C
2. (A • B)
3. ∴ C

In this argument we can assert C according to the rule, modus ponens. This is so even though the antecedent of the conditional is itself complex (i.e., it is a conjunction). That doesn't matter. The first premise is still a conditional statement (since the horseshoe is the main operator) and the second premise is the antecedent of that conditional statement. The rule modus ponens says that if we have that much, we are entitled to infer the consequent of the conditional.

We can actually use modus ponens in the first argument of this section:

1. (R v S) ⊃ (T ⊃ K)
2. ~K
3. R v S /∴ ~T

4. T ⊃ K Modus ponens, lines 1, 3

What I have done here is I have written the valid form of inference (or rule) that justifies the line I am deriving, as well as the lines to which that rule applies, to the right of the new line of the proof that I am deriving. Here I have derived "T ⊃ K" from lines 1 and 3 of the argument by modus ponens. Notice that line 1 is a conditional statement and line 3 is the antecedent of that conditional statement. This proof isn't finished yet, since we have not yet derived the conclusion we are trying to derive, namely, "~T." We need a different rule to derive that, which we will introduce next.

The next form of inference is called "**modus tollens**," which is Latin for "the way that denies." Modus tollens has the following form:

1. p ⊃ q
2. ~q
3. ∴ ~p

What this form says, in words, is that if we have asserted a conditional statement (p ⊃ q) and we have also asserted the negated consequent of that conditional (~q), then we are entitled to infer the negated antecedent of that conditional statement (~p). For example, if I asserted the conditional, "if it is raining, then the ground is wet" and I also asserted "the ground is not wet" (the negated consequent of that conditional) then I am entitled to assert the negated antecedent of the conditional, "it is not raining." It is important to see that *any* argument that has *this same form* is a valid argument. For example, the following argument is also an argument with this same form:

1. C ⊃ (E v F)
2. ~(E v F)
3. ∴ ~C

In this argument we can assert ~C according to the rule, modus tollens. This is so even though the consequent of the conditional is itself complex (i.e., it is a disjunction). That doesn't matter. The first premise is still a conditional statement (since the horseshoe is the main operator) and the second premise is the negated consequent of that conditional statement. The rule modus tollens says that if we have that much, we are entitled to infer the negated antecedent of the conditional.

We can use modus tollens to complete the proof we started above:

1. (R v S) ⊃ (T ⊃ K)
2. ~K
3. R v S /∴ ~T
4. T ⊃ K Modus ponens, lines 1, 3
5. ~T Modus tollens, lines 2, 4

Notice that the last line of the proof is the conclusion that we are supposed to derive and that each statement that I have derived (i.e., lines 4 and 5) has a rule to the right. That rule cited is the rule that justifies the statement that is being derived and the lines cited are the previous lines of the proof where we can see that the rule applies. This is what is called a proof. A **proof** is a series of statements, starting with the premises and ending with the conclusion, where each additional statement after the premises is derived from some previous line(s) of the proof using one of the valid forms of inference. We will practice this some more in the exercise at the end of this section.

The next form of inference is called "**hypothetical syllogism**." This is what ancient philosophers called "the chain argument" and it should be obvious why in a moment. Here is the form of the rule:

1. p ⊃ q
2. q ⊃ r
3. ∴ p ⊃ r

As you can see, the conclusion of this argument links p and r together in a conditional statement. We could continue adding conditionals such as "r ⊃ s" and "s ⊃ t" and the inferences would be just as valid. And if we lined them all up as I have below, you can see why ancient philosophers referred to this valid argument form as a "chain argument":

p ⊃ q
q ⊃ r
r ⊃ s
s ⊃ **t**
∴ p ⊃ t

Notice how the consequent of each preceding conditional statement links up with the antecedent of the next conditional statement in such a way as to create a chain. The chain could be as long as we liked, but the rule that we will cite in our proofs only connects two different conditional statements together. As before, it is important to realize that any argument with this same form is a valid argument. For example,

1. (A v B) ⊃ ~D
2. ~D ⊃ C
3. ∴ (A v B) ⊃ C

Notice that the consequent of the first premise and the antecedent of the second premise are exactly the same term, "~D". That is what allows us to "link" the antecedent of the first premise and the consequent of the second premise together in a "chain" to infer the conclusion. Being able to recognize the forms of these inferences is an important skill that you will have to become proficient at in order to do proofs.

The next four forms of inference we will introduce utilize conjunction, disjunction and negation in different ways. We will start with the rule called "**simplification**," which has the following form:

1. p • q
2. ∴ p

What this rule says, in words, is that if we have asserted a conjunction then we are entitled to infer either one of the conjuncts. This is the rule that I introduced in the first section of this chapter. It is a pretty "obvious" rule—so obvious, in fact, that we might even wonder why we have to state it. However, every form of inference that we will introduce in this section should be obvious—that is the point of calling them *basic* forms of inference. They are some of the simplest forms of inference, whose validity should be transparently obvious. The idea of a proof is that although the inference being made in the argument is *not* obvious, we can break that inference down in steps, each of which *is* obvious. Thus, the obvious inferences ultimately justify the non-obvious inference being made in the argument. Those obvious inferences thus function as *rules* that we use to justify each step of the proof. Simplification is a prime example of one of the more obvious rules.

As before, it is important to realize that any inference that has the same form as simplification is a valid inference. For example,

1. (A v B) · ~(C · D)
2. ∴ (A v B)

is a valid inference because it has the same form as simplification. That is, line 1 is a conjunction (since the dot is the main operator of the sentence) and line 2 is inferring one of the conjuncts of that conjunction in line 1. (Just think of the "A v B" as the "p" and the "~(C · D)" as the "q".)

The next rule we will introduce is called "**conjunction**" and is like the reverse of simplification. (Don't confuse the *rule* called conjunction with the *type of complex proposition* called a conjunction.) Conjunction has the following form:

1. p
2. q
3. ∴ p · q

What this rule says, in words, is that if you have asserted two different propositions, then you are entitled to assert the conjunction of those two propositions. As before, it is important to realize that any inference that has the same form as conjunction is a valid inference. For example,

1. A ⊃ B
2. C v D
3. ∴ (A ⊃ B) · (C v D)

is a valid inference because it has the same form as conjunction. We are simply conjoining two propositions together; it doesn't matter whether those propositions are atomic or complex. In this case, of course, the propositions we are conjoining together are complex, but as long as those propositions have already been asserted as premises in the argument (or derived by some other valid form of inference), we can conjoin them together into a conjunction.

The next form of inference we will introduce is called "**disjunctive syllogism**" and it has the following form:

1. p v q

2. ~p
3. ∴ q

In words, this rule states that if we have asserted a disjunction and we have asserted the negation of one of the disjuncts, then we are entitled to assert the other disjunct. Once you think about it, this inference should be pretty obvious. If we are taking for granted the truth of the premises—that either p or q is true; and that p is not true—then is *has to follow* that q is true in order for the original disjunction to be true. (Remember that we *must* assume the premises are true when evaluating whether an argument is valid.) If I assert that it is true that either Bob or Linda stole the diamond, and I assert that Bob did not steal the diamond, then it has to follow that Linda did. That is a disjunctive syllogism. As before, any argument that has this same form is a valid argument. For example,

1. ~A v (B ⋅ C)
2. ~~A
3. ∴ B ⋅ C

is a valid inference because it has the same form as disjunctive syllogism. The first premise is a disjunction (since the wedge is the main operator), the second premise is simply the negation of the left disjunct, "~A", and the conclusion is the right disjunct of the original disjunction. It may help you to see the form of the argument if you treat "~A" as the p and "B ⋅ C" as the q. Also notice that the second premise contains a double negation. Your English teacher may tell you never to use double negatives, but as far as logic is concerned, there is absolutely nothing wrong with a double negation. In this case, our left disjunct in premise 1 is itself a negation, while premise 2 is simply a negation of that left disjunct.

The next rule we'll introduce is called "**addition**." It is not quite as "obvious" a rule as the ones we've introduced above. However, once you understand the conditions under which a disjunction is true, then it should be obvious why this form of inference is valid. Addition has the following form:

1. p
2. ∴ p v q

What this rule says, in words, is that that if we have asserted some proposition, p, then we are entitled to assert the disjunction of that proposition p and *any* *other proposition q we wish*. Here's the simple justification of the rule. If we know that p is true, and a disjunction is true if at least one of the disjuncts is true, then we know that the disjunction p v q is true even if we don't know whether q is true or false. Why? Because *it doesn't matter* whether q is true or false, since we already know that p is true. The hardest thing to understand about this rule is why we would ever want to use it. The best answer I can give you for that right now is that it can help us out when doing proofs.[3]

As before, is it important to realize that any argument that has this same form, is a valid argument. For example,

1. A v B
2. ∴ (A v B) v (~C v D)

is a valid inference because it has the same form as addition. The first premise asserts a statement (which in this case is complex—a disjunction) and the conclusion is a disjunction of that statement and *some other statement*. In this case, that other statement is itself complex (a disjunction). But an argument or inference can have the same form, regardless of whether the components of those sentences are atomic or complex. That is the important lesson that I have been trying to drill in in this section.

The final of our 8 valid forms of inference is called "**constructive dilemma**" and is the most complicated of them all. It may be most helpful to introduce it using an example. Suppose I reasoned thus:

> The killer is either in the attic or the basement. If the killer is in the attic then he is above me. If the killer is in the basement then his is below me. Therefore, the killer is either _________________ or _________________.

Can you fill in the blanks with the phrases that would make this argument valid? I'm guessing that you can. It should be pretty obvious. The conclusion of the argument is the following:

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[3] A better answer is that we need this rule in order to make this set of rules that I am presenting a sound a complete set of rules. That is, without it there would be arguments that are valid but that we aren't able to show are valid using this set of rules. In more advanced areas of logic, such as metalogic, logicians attempt to prove things about a particular system of logic, such as proving that the system is sound and complete.

The killer is either above me or below me.

That this argument is valid should be obvious (can you imagine a scenario where all the premises are true and yet the conclusion is false?). What might not be as obvious is the form that this argument has. However, you should be able to identify that form if you utilize the tools that you have learned so far. The first premise is a disjunction. The second premise is a conditional statement whose antecedent is the left disjunct of the disjunction in the first premise. And the third premise is a conditional statement whose antecedent is the right disjunct of the disjunction in the first premise. The conclusion is the disjunction of the consequents of the conditionals in premises 2 and 3. Here is this form of inference using symbols:

1. p v q
2. p ⊃ r
3. q ⊃ s
4. ∴ r v s

We have now introduced each of the 8 forms of inference. In the next section I will walk you through some basic proofs that utilize these 8 rules.

Exercise 16: Fill in the blanks with the valid form of inference that is being used and the lines the inference follows from. Note: the conclusion is written to the right of the last premise, following the "/∴" symbols.

Example 1:

1. M ⊃ ~N
2. M
3. H ⊃ N   /∴ ~H
4. ~N   Modus ponens, 1, 2
5. ~H   Modus tollens, 3, 4

Example 2:

1. A v B
2. C ⊃ D
3. A ⊃ C
4. ~D   /∴ B
5. A ⊃ D   Hypothetical syllogism, 3, 2

6. ~A <u>Modus tollens, 5, 4</u>
7. B <u>Disjunctive syllogism, 1, 6</u>

1

1. A • C /∴ (A v E) • (C v D)
2. A ________________
3. C ________________
4. A v E ________________
5. C v D ________________
6. (A v E) • (C v D) ________________

2

1. A ⊃ (B ⊃ D)
2. ~D
3. D v A /∴ ~B
4. A ________________
5. B ⊃ D ________________
6. ~B ________________

3

1. A ⊃ ~B
2. A v C
3. ~~B • D /∴ C
4. ~~B ________________
5. ~A ________________
6. C ________________

#4

1. A ⊃ B
2. A • ~D
3. B ⊃ C /∴ C • ~D
4. A ________________
5. A ⊃ C ________________
6. C ________________
7. ~D ________________
8. C • ~D ________________

#5

1. C
2. A ⊃ B
3. C ⊃ D
4. D ⊃ E /∴ E v B
5. C ⊃ E ________________
6. C v A ________________
7. E v B ________________

#6

1. (A v M) ⊃ R
2. (L ⊃ R) • ~R
3. ~(C • D) v (A v M) /∴ ~(C • D)
4. ~R ________________
5. ~(A v M) ________________
6. ~(C • D) ________________

#7

1. (H • K) ⊃ L
2. ~R • K
3. K ⊃ (H v R) /∴ L
4. K ________________
5. H v R ________________
6. ~R ________________
7. H ________________
8. H • K ________________
9. L ________________

#8

1. C ⊃ B
2. ~D • ~B
3. (A ⊃ (B ⊃ C)) v D
4. A v C /∴ B ⊃ C
5. ~D ________________
6. A ⊃ (B ⊃ C) ________________
7. ~B ________________
8. ~C ________________
9. A ________________
10. B ⊃ C ________________
11. (B ⊃ C) v B ________________

2.12 How to construct proofs

You can think of constructing proofs as a game. The goal of the game is to derive the conclusion from the given premises using only the 8 valid rules of inference that we have introduced. Not every proof requires you to use every rule, of course. But you may use any of the rules—as along as your use of the rule is correct. Like most games, people can be better or worse at the "game" of constructing proofs. Better players will be able to a) make fewer mistakes, b) construct the proofs more quickly, and c) construct the proofs more efficiently. In order to construct proofs, it is imperative that you internalize the 8 valid forms of inference introduced in the previous section. You will be citing these forms of inference as rules that will justify each new line of your proof that you add. By "internalize" I mean that you have memorized them so well that you can see those forms manifest in various sentences almost without even thinking about it. If you internalize the rules in this way, constructing proofs will be a pleasant diversion, rather than a frustrating activity. In addition to internalizing the 8 valid forms of inference, there are a couple of different strategies that can help when you're stuck and can't figure out what to do next. The first is the **strategy of working backwards**. When we work backwards in a proof, we ask ourselves what rule we can use to derive the sentence(s) we need to derive. Here is an example:

1. R · S
2. T /∴ (T v L) · (R · S)

The conclusion, which is to the right of the second premise and follows the "/∴" symbol, is a conjunction (since the dot is the main operator). If we are trying to "work backwards," the relevant question to ask is: What rule can we use to derive a conjunction? If you know the rules, you should know the answer to that question. There is only one rule that allows us to derive (infer) a sentence that is a conjunction. That rule is called "conjunction." The form of the rule conjunction says that in order to derive a conjunction, we need to have each conjunct on a separate line. So, what are the two conjuncts that we would need in order to derive the conjunction that is the conclusion (i.e., "(T v L) · (R · S)"). We would need both "T v L" on a line and "R · S" on a separate line. But look at premise 1—we already have "R · S" on its own line! So the only other thing we need to derive is the sentence "T v L". Once we have that on a separate line, then we can use the rule conjunction to conjoin those two sentences to get the conclusion! So the next question we have to ask is: How can I derive the sentence "T v L"? Again, if we are working backwards, the relevant question to ask here is: What rule allows me to derive a disjunction? There are only two: constructive dilemma and addition. However, we know that we won't be using constructive dilemma since none of the premises are conditional statements, and constructive dilemma requires conditional statements as premises. That leaves addition. Addition allows us to disjoin any statement we like to an existing statement. Since we have "T" as the second premise, the rule addition allows us to disjoin "L" to that statement. The first new line of the proof should thus look like this:

3. T v L Addition 2

What I have done is number a new line of the proof (continuing the numbering from the premises) and then have written the rule that justifies that new line as well as the line(s) from which that line was derived via that rule. In this case, since addition is a rule that allows you to derive a sentence directly from just one line, I have cited only one line. The next step of the proof should be clear since we have already talked through it above. All we have to do now is go directly to the conclusion, since the conclusion is a conjunction and we now have (on separate lines of the proof) each conjunct. Thus, the final line of this (quite simple) proof should look like this:

4. (T v L) • (R • S) Conjunction 1, 3

Again, all I've done is the write the new line of the proof (continuing the numbering from the previous line) and then have written the rule that justifies that new line as well as the line(s) from which that line was derived via that rule. In this case, the rule conjunction requires that we cite two lines (i.e., each conjunct that we are conjoining). So, I have to find the lines that contained "T v L" and "R • S" and cite those lines. It does not matter the order in which you cite the lines as along as you have cited the correct lines (e.g., I could have equally well have written, "Conjunction 3, 1" as the justification). Thus the complete proof should look like this:

1. R • S
2. T /∴ (T v L) • (R • S)
3. T v L Addition 2

4. (T v L) · (R · S)   Conjunction 1, 3

That's it. That is all there is to constructing a proof. The last line of the proof is the conclusion to be derived: check. Each line of the proof follows by the rule and the line(s) cited: check. Since both of those requirements check out, our proof is complete and correct.

I have just walked you through a simple proof using the strategy of working backwards. This strategy works well as long as the conclusion we are trying to derive is complex—that is, if it contains truth functional connectives. However, sometimes our conclusion will simply be an atomic statement. In that case, we will not as easily be able to utilize the strategy of working backwards. But there is another strategy that we can utilize: the **strategy of working forward**. To utilize the strategy of working forward, we simply ask ourselves what rules we can apply to the existing premises to derive *something*, even if it isn't the conclusion we are ultimately trying to derive. As a part of this strategy, we should typically break apart a conjunction whenever we have one as a premise of our argument. Doing this can help to see where to go next. (If you've ever played Scrabble, then you can think of this as rearranging your Scrabble tiles in order to see what words you can build.) Here is an example of a proof where we should utilize the strategy of working forward:

1. A · B
2. B ⊃ C   /∴ C

Notice that since the conclusion is atomic, we cannot utilize the strategy of working backwards. Instead, we should try working forward. As part of this strategy, we should break apart conjunctions by using the rule "simplification." That will be the first step of our proof:

1. A · B
2. B ⊃ C   /∴ C
3. A   Simplification 1
4. B   Simplification 1

The first two lines of the proof is just breaking down the conjunction in line 1, where line 3 is just the left conjunct and line 4 is just the right conjunct. Both lines 3 and 4 follow by the same rule and the same line, in this case. The next question we ask when utilizing the strategy of working forward is: what lines of the proof we can apply some rule to in order to derive *something or other*? Look at the conditional on line 2. We haven't used that yet. So what rule can we apply to that line? You should be thinking of the rules that utilize conditional statements (modus ponens, modus tollens, and hypothetical syllogism). We can rule out hypothetical syllogism since here we have only *one* conditional and the rule hypothetical syllogism requires that we have two. If you look at line 4 (that we have just derived) you should see that it is the antecedent of the conditional statement on line 2. And you should know that that means we can apply the rule, modus ponens. So our next step is to do that:

1. A • B
2. B ⊃ C /∴ C
3. A Simplification 1
4. B Simplification 1
5. C Modus ponens 2, 4

But now also notice that the line that we have just derived is in fact the conclusion of the argument. So our proof is finished.

Before the close of this section, let's work through a bit longer proof. Remember: any proof, long or short, is the same process and utilizes the same strategy. It is just a matter a keeping track of where you are in the proof and what you're ultimately trying to derive. So here is a bit more complex proof:

1. (~A v B) ⊃ L
2. ~B
3. A ⊃ B
4. L ⊃ (~R v D)
5. ~D • (R v F) /∴ (L v G) • ~R

The conclusion is a conjunction of "L v G" and "~R" so we know that if we can get each of those sentences on a separate line, then we can use the rule conjunction to derive the conclusion. That will be our long range goal here (and this is utilizing the strategy of working backwards). However, we cannot see how to directly get there from here at this point, so we will begin utilizing the strategy of working forward. The first thing we'll do is simplify the conjunction on line 5:

6. ~D Simplification 5

7. R v F Simplification 5

Look at lines 2 and 6: they are both negated atomic propositions. Another part of the strategy of working forward is to utilize either atomic or negated atomic sentences. We should look for how we can utilize modus tollens or disjunctive syllogism by plugging these negated atomic sentences into other lines of the proof. Look at lines 2 and 3. You should see a modus tollens there. That will be our next step:

8. ~A Modus tollens 2, 3

The next step of this proof can be a bit tricky. There are a couple different ways we could go. One would be to utilize the rule "addition." Can you see how we might helpfully utilize this rule using either line 6 or 8? If not, I'll give you a hint: what if we were to use addition on line 8 to derive "~A v B"? Can you see how we could then plug that into line 1? In fact, "~A v B" is the antecedent of the conditional in line 1, so we could then use modus ponens to derive the consequent. Thus, let's try starting with the addition on line 8:

9. ~A v B Addition 8

Next, we'll utilize line 9 and line 1 with modus ponens to derive the next line:

10. L Modus ponens 1, 9

Notice at this point that what we have derived on line 10 is "L" and what we earlier said we needed as one of the conjuncts was "L v G". You should recognize that we have a rule that will allow us to infer directly from "L" to "L v G". That rule is addition (again). That will be the next line of the proof:

11. L v G Addition 10

At this point, our strategy should be to try to derive the other conjunct, "~R". Notice that "~R" is contained within the sentence on line 4, but it is embedded. How can we "get it free"? Start by noticing that the ~R is a part of a disjunction, which is itself a consequent of a conditional statement. Also notice that we have already derived the antecedent of that conditional statement, which means that we can use modus ponens to derive the consequent:

12. ~R v D Modus ponens 4, 10

The penultimate step is to use a disjunctive syllogism to derive "~R".

13. ~R Disjunctive syllogism 6, 12

The final step is simply to conjoin lines 11 and 13 to get the conclusion:

14. (L v G) • ~R Conjunction 11, 13

Thus, here is the completed proof:

1. (~A v B) ⊃ L
2. ~B
3. A ⊃ B
4. L ⊃ (~R v D)
5. ~D • (R v F) /∴ (L v G) • ~R
6. ~D Simplification 5
7. R v F Simplification 5
8. ~A Modus tollens 2, 3
9. ~A v B Addition 8
10. L Modus ponens 1, 9
11. L v G Addition 10
12. ~R v D Modus ponens 4, 10
13. ~R Disjunctive syllogism
14. (L v G) • ~R Conjunction 11, 13

Constructing proofs is a skill that takes practice. The following exercises will give you some practice with constructing proofs.

Exercise 17: Construct proofs for the following valid arguments. The first fifteen proofs *can* be complete in three or less additional lines. The next five proofs will be a bit longer. It is important to note that there is always more than one way to construct a proof. If your proof differs from the answer key, that doesn't mean it is wrong.

#1

1. A • B
2. (A v C) ⊃ D /∴ A • D

#2
1. A
2. B /∴ (A v C) ・ B

#3
1. D ⊃ E
2. D ・ F /∴ E

#4
1. J ⊃ K
2. J /∴ K v L

#5
1. A v B
2. ~A ・ ~C /∴ B

#6
1. A ⊃ B
2. ~B ・ ~C /∴ ~A

#7
1. D ⊃ E
2. (E ⊃ F) ・ (F⊃ D) /∴D ⊃ F

#8
1. (T ⊃ U) ・ (T ⊃ V)
2. T /∴ U v V

#9
1. (E ・ F) v (G ⊃ H)
2. I ⊃ G
3. ~(E ・ F) /∴ I ⊃ H

#10
1. M ⊃ N
2. O ⊃ P
3. N ⊃ P
4. (N ⊃ P) ⊃ (M v O) /∴N v P

#11
1. A v (B ⊃ A)
2. ~A ・ C /∴ ~B

#12
1. (D v E) ⊃ (F ・ G)
2. D /∴ F

#13
1. T ⊃ U
2. V v ~U
3. ~V ・ ~W /∴ ~T

#14
1. (A v B) ⊃ ~C
2. C v D
3. A /∴ D

#15
1. L v (M ⊃ N)
2. ~L ⊃ (N ⊃ O)
3. ~L /∴ M ⊃ O

#16
1. A ⊃ B
2. A v (C ・ D)
3. ~B ・ ~E /∴ C

#17
1. (F ⊃ G) ・ (H ⊃ I)
2. J ⊃ K
3. (F v J) ・ (H v L) /∴ G v K

#18
1. (E v F) ⊃ (G ・ H)
2. (G v H) ⊃ I
3. E /∴ I

#19
1. (N v O) ⊃ P
2. (P v Q) ⊃ R
3. Q v N
4. ~Q /∴ R

#20

1. J ⊃ K
2. K v L
3. (L • ~J) ⊃ (M • ~J)
4. ~K /∴ M